5.6 Direct Variation

Essential Question How can you use a graph to show the relationship between two quantities that vary directly? How can you use an equation?

1 ACTIVITY: Math in Literature

Gulliver's Travels was written by Jonathan Swift and published in 1726. Gulliver was shipwrecked on the island Lilliput, where the people were only 6 inches tall. When the Lilliputians decided to make a shirt for Gulliver, a Lilliputian tailor stated that he could determine Gulliver's measurements by simply measuring the distance around Gulliver's thumb. He said "Twice around the thumb equals once around the wrist. Twice around the wrist is once around the neck. Twice around the neck is once around the waist."

Work with a partner. Use the tailor's statement to complete the table.

Thumb, t	Wrist, w	Neck, n	Waist, x
0 in.			
1 in.			
	4 in.		
		12 in.	
			32 in.
	10 in.		

COMMON CORE

Direct Variation

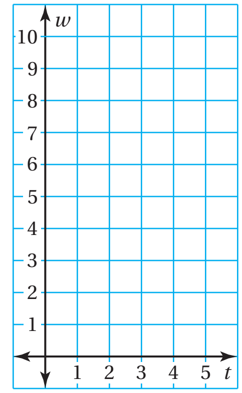
In this lesson, you will

- identify direct variation from graphs or equations.
- use direct variation models to solve problems.

Learning Standards
7.RP.2a
7.RP.2b
7.RP.2c
7.RP.2d

2 ACTIVITY: Drawing a Graph

Work with a partner. Use the information from Activity 1.



a. In your own words, describe the relationship between *t* and *w*.

b. Use the table to write the ordered pairs (*t*, *w*). Then plot the ordered pairs.

c. What do you notice about the graph of the ordered pairs?

d. Choose two points and find the slope of the line between them.

e. The quantities *t* and *w* are said to *vary directly*. An equation that describes the relationship is

$$w = \square\, t.$$

3 ACTIVITY: Drawing a Graph and Writing an Equation

Math Practice 6

Label Axes

How do you know which labels to use for the axes? Explain.

Work with a partner. Use the information from Activity 1 to draw a graph of the relationship. Write an equation that describes the relationship between the two quantities.

a. Thumb *t* and neck *n* ($n = \square\, t$)

b. Wrist *w* and waist *x* ($x = \square\, w$)

c. Wrist *w* and thumb *t* ($t = \square\, w$)

d. Waist *x* and wrist *w* ($w = \square\, x$)

What Is Your Answer?

4. **IN YOUR OWN WORDS** How can you use a graph to show the relationship between two quantities that vary directly? How can you use an equation?

5. **STRUCTURE** How are all the graphs in Activity 3 alike?

6. Give a real-life example of two variables that vary directly.

7. Work with a partner. Use string to find the distance around your thumb, wrist, and neck. Do your measurements agree with the tailor's statement in *Gulliver's Travels*? Explain your reasoning.

Practice

Use what you learned about quantities that vary directly to complete Exercises 4 and 5 on page 202.

5.6 Lesson

Check It Out
Lesson Tutorials
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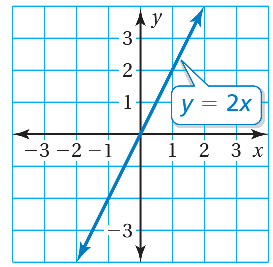
Key Vocabulary
direct variation, *p. 200*
constant of proportionality, *p. 200*

Key Idea

Direct Variation

Words Two quantities x and y show **direct variation** when $y = kx$, where k is a number and $k \neq 0$. The number k is called the **constant of proportionality**.

Graph The graph of $y = kx$ is a line with a slope of k that passes through the origin. So, two quantities that show direct variation are in a proportional relationship.



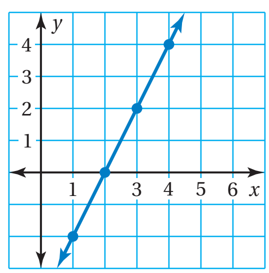
EXAMPLE 1 Identifying Direct Variation

Tell whether x and y show direct variation. Explain your reasoning.

a.

x	1	2	3	4
y	−2	0	2	4

Plot the points. Draw a line through the points.

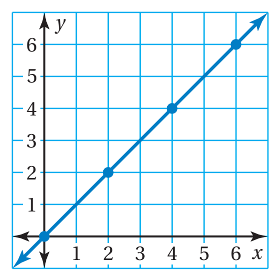


The line does *not* pass through the origin. So, x and y do *not* show direct variation.

b.

x	0	2	4	6
y	0	2	4	6

Plot the points. Draw a line through the points.



The line passes through the origin. So, x and y show direct variation.

Study Tip

Other ways to say that x and y show direct variation are "y varies directly with x" and "x and y are directly proportional."

EXAMPLE 2 Identifying Direct Variation

Tell whether x and y show direct variation. Explain your reasoning.

a. $y + 1 = 2x$

$y = 2x - 1$ Solve for y.

The equation *cannot* be written as $y = kx$. So, x and y do *not* show direct variation.

b. $\frac{1}{2}y = x$

$y = 2x$ Solve for y.

The equation can be written as $y = kx$. So, x and y show direct variation.

Multi-Language Glossary at BigIdeasMath.com

On Your Own

Now You're Ready
Exercises 6–17

Tell whether *x* and *y* show direct variation. Explain your reasoning.

1.

x	y
0	−2
1	1
2	4
3	7

2.

x	y
1	4
2	8
3	12
4	16

3.

x	y
−2	4
−1	2
0	0
1	2

4. $xy = 3$

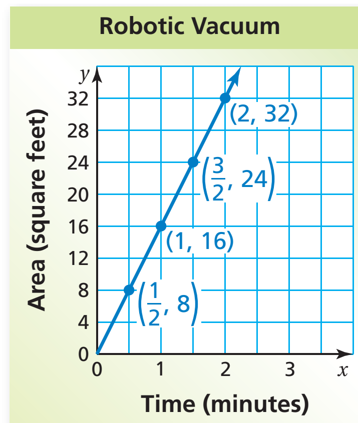
5. $x = \frac{1}{3}y$

6. $y + 1 = x$

EXAMPLE 3 Real-Life Application

x	y
$\frac{1}{2}$	8
1	16
$\frac{3}{2}$	24
2	32

The table shows the area *y* (in square feet) that a robotic vacuum cleans in *x* minutes.



a. Graph the data. Tell whether *x* and *y* are directly proportional.

Graph the data. Draw a line through the points.

The graph is a line through the origin. So, x and y are directly proportional.

b. Write an equation that represents the line.

Choose any two points to find the slope of the line.

$$\text{slope} = \frac{\text{change in } y}{\text{change in } x} = \frac{16}{1} = 16$$

The slope of the line is the constant of proportionality, k. So, an equation of the line is $y = 16x$.

c. Use the equation to find the area cleaned in 10 minutes.

$y = 16x$	Write the equation.
$= 16(10)$	Substitute 10 for x.
$= 160$	Multiply.

So, the vacuum cleans 160 square feet in 10 minutes.

On Your Own

Now You're Ready
Exercise 19

7. **WHAT IF?** The battery weakens and the robot begins cleaning less and less area each minute. Do x and y show direct variation? Explain.

5.6 Exercises

Check It Out
Help with Homework
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Vocabulary and Concept Check

1. **VOCABULARY** What does it mean for x and y to vary directly?

2. **WRITING** What point is on the graph of every direct variation equation?

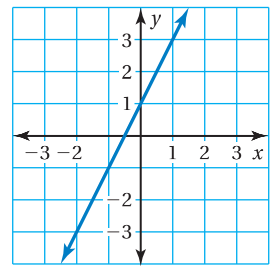
3. **DIFFERENT WORDS, SAME QUESTION** Which is different? Find "both" answers.

Do x and y show direct variation?

Are x and y in a proportional relationship?

Is the graph of the relationship a line?

Does y vary directly with x?



Practice and Problem Solving

Graph the ordered pairs in a coordinate plane. Do you think that graph shows that the quantities vary directly? Explain your reasoning.

4. $(-1, -1), (0, 0), (1, 1), (2, 2)$

5. $(-4, -2), (-2, 0), (0, 2), (2, 4)$

Tell whether x and y show direct variation. Explain your reasoning. If so, find k.

1 6.

x	1	2	3	4
y	2	4	6	8

7.

x	−2	−1	0	1
y	0	2	4	6

8.

x	−1	0	1	2
y	−2	−1	0	1

9.

x	3	6	9	12
y	2	4	6	8

2 10. $y - x = 4$

11. $x = \frac{2}{5}y$

12. $y + 3 = x + 6$

13. $y - 5 = 2x$

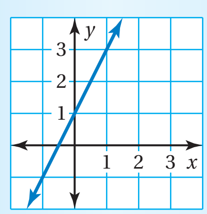
14. $x - y = 0$

15. $\frac{x}{y} = 2$

16. $8 = xy$

17. $x^2 = y$

18. **ERROR ANALYSIS** Describe and correct the error in telling whether x and y show direct variation.



The graph is a line, so it shows direct variation.

3 19. **RECYCLING** The table shows the profit y for recycling x pounds of aluminum. Graph the data. Tell whether x and y show direct variation. If so, write an equation that represents the line.

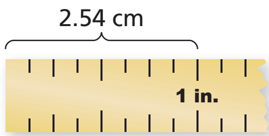
Aluminum (lb), x	10	20	30	40
Profit, y	\$4.50	\$9.00	\$13.50	\$18.00

The variables x and y vary directly. Use the values to find the constant of proportionality. Then write an equation that relates x and y.

20. $y = 72; x = 3$

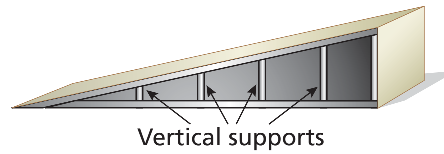
21. $y = 20; x = 12$

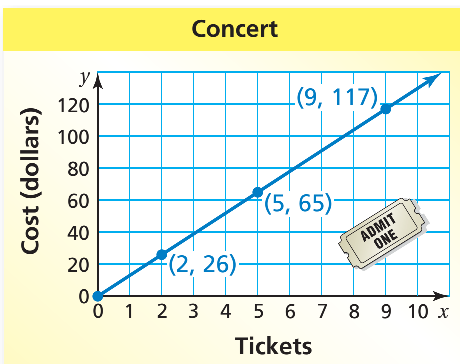
22. $y = 45; x = 40$



23. **MEASUREMENT** Write a direct variation equation that relates x inches to y centimeters.

24. **MODELING** Design a waterskiing ramp. Show how you can use direct variation to plan the heights of the vertical supports.





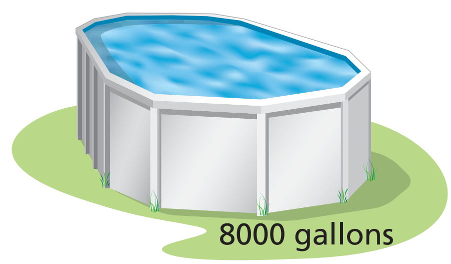
25. **REASONING** Use $y = kx$ to show why the graph of a proportional relationship always passes through the origin.

26. **TICKETS** The graph shows the cost of buying concert tickets. Tell whether x and y show direct variation. If so, find and interpret the constant of proportionality. Then write an equation and find the cost of 14 tickets.

27. **CELL PHONE PLANS** Tell whether x and y show direct variation. If so, write an equation of direct variation.

Minutes, x	500	700	900	1200
Cost, y	\$40	\$50	\$60	\$75

28. **CHLORINE** The amount of chlorine in a swimming pool varies directly with the volume of water. The pool has 2.5 milligrams of chlorine per liter of water. How much chlorine is in the pool?



29. **Critical Thinking** Is the graph of every direct variation equation a line? Does the graph of every line represent a direct variation equation? Explain your reasoning.

Fair Game Review *What you learned in previous grades & lessons*

Write the fraction as a decimal. *(Section 2.1)*

30. $\frac{13}{20}$

31. $\frac{9}{16}$

32. $\frac{21}{40}$

33. $\frac{24}{25}$

34. **MULTIPLE CHOICE** Which rate is *not* equivalent to 180 feet per 8 seconds? *(Section 5.1)*

Ⓐ $\frac{225 \text{ ft}}{10 \text{ sec}}$

Ⓑ $\frac{45 \text{ ft}}{2 \text{ sec}}$

Ⓒ $\frac{135 \text{ ft}}{6 \text{ sec}}$

Ⓓ $\frac{180 \text{ ft}}{1 \text{ sec}}$